# MAT 230 Module Five Homework

**General:**

* Before beginning this homework, be sure to read the textbook sections and the material in Module Five.
* Type your solutions into this document and be sure to show all steps for arriving at your solution. Just giving a final number may not receive full credit.
* You may copy and paste mathematical symbols from the statements of the questions into your solution. This document was created using the Arial Unicode font.
* These homework problems are proprietary to SNHU COCE. They may not be posted on any non-SNHU website.
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1. For A = {a, b, c} and B = {5, 10, 15, 20}.
   1. How many elements are in A × B?
   2. List the elements of A × B.

This problem is similar to Example 4 and to Exercises 5–7 in Section 4.1 of your SNHU MAT230 textbook.

1. 12
2. A x B = {(a, 5), (b, 5), (c, 5), (a, 10), (b, 10), (c, 10), (a, 15), (b, 15), (c, 15), (a, 20), (b, 20), (c, 20)
3. Let A = ℤ+, the positive integers, and let R be the relation defined by  
   a R b if and only if 3a < 2b + 5.
   1. Give two ordered pairs that belong to R.
   2. Give two ordered pairs that do not belong to R.

This problem is similar to Examples 3 and 4 and to Exercises 1–3 in Section 4.2 of your SNHU MAT230 textbook.

1. (1, 2), (2, 3)
2. (10, 1), (100, 2)
3. Let A = {1, 2, 3, 4, 5, 6} = B. Define a relation R as a R b if and only if a + b < 6. Find the domain, range, matrix representation of R, and the digraph of R. You may use (copy/paste/move/resize/etc.) the images below to create your graph.

This problem is similar to Examples 11, 19, and 23 and to Exercises 10–12 in Section 4.2 of your SNHU MAT230 textbook.

R = {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (3,1), (3,2), (4,1)}

Dom(R) = {1, 2, 3, 4}  
Ran(R) = {1, 2, 3, 4}

Matrix Representation:

Digraph:

1. Determine whether the relation R defined below is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. For each property, either explain why R has that property or give an example showing why it does not.
   1. Let A = {1, 2, 3, 4} and let R = { (2, 3) }
   2. Let A = {1, 2, 3, 4} and let R = { (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 4), (3, 1), (3, 3), (4, 1), (4, 4) }.

This problem is similar to Examples 1, 4, and 10 and to Exercises 1–4, 7, and 8 in Section 4.4 of your SNHU MAT230 textbook.

1. Irreflexive because (a,a) is not in the set of R for all of a in the set of A. Asymmetric because (a,b) in the set of R implies (b, a) is not in the set of R, Transitive because it cannot be proven otherwise given the lack of c.

If it were reflexive, we would see (a,a) in the set of R for all of A in the set of A. For example (1,1), (2,2), (3,3), (4,4) would all be in the set of R.

If it were symmetric, we would see for all (a,b) in the set of R that (b,a) was also in the set of R. For example, we would also see (3,2) in this set. It is not.

If it were antisymmetric, we would see (a,b) AND (b,a) in this set (already disproven) and that a = b.

1. Reflexive because (a,a) is in the set of R for all of a in the set of A.

If it were irreflexive there would be no instance of (a,a) in the set of R. This is not the case.

If it were symmetric, for each (a,b) in R, we would also find (b,a). We would need to add two elements to our relation in order for this to be true: (4,2) and (1,4).

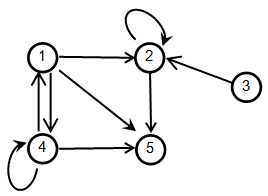
If it were asymmetric, for each (a,b) in R we could not find (b,a) in R. This is not the case as we have (2,1) and (1,2) for example.

If it were antisymmetric for each (a,b) in R we would also find (b,a) (already disproven) and it would mean that a = b which is false for most of our elements.

If it were transitive, for all elements (a,b) in the set of R and (b,c) in the set of R, we would also see that (a,c) was in the set of R. For example, we have (1,2) and (2,4), but we do not have (1,4) so it is not a transitive set.

1. Let A = {1, 2, 3, 4, 5} and R be the relation on the set A whose digraph is shown below. Determine whether R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. For each property, either explain why R has that property or give an example showing why it does not.

This problem is similar to a combination of Example 23 in Section 4.2 with Example 6 in Section 4.4, and to Exercises 9 and 10 in Section 4.4 of your SNHU MAT230 textbook.



R = {(1,2), (1,4), (1,5), (2,2), (2,5), (3,2), (4,1), (4,5)}

Reflexive – No because every element does not have a loop. (a,a)

Irreflective – No, because every element would need to not have a loop.

Symmetric – No, because every element would need to have a double arrow.

Asymmetric – No, because every edge would have a double arrow and no loops.

Antisymmetric – No, because no edge would have a double arrow.

Transitive – Yes, because for every case that we can test (a,b) and (b,c) in the set of R, (a,c) exists in R.

1. Let A = {1, 2, 3, 4, 5} and R be the relation on the set A whose matrix is shown below. Determine whether R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. For each property, either explain why R has that property or give an example showing why it does not.

This problem is similar to Examples 6 and 11 and to Exercises 11 and 12 in Section 4.4 of your SNHU MAT230 textbook.



R = {(1,1), (1,2), (2,1), (2,2), (2,5), (3,3), (3,5), (5,2), (5,3), (5,5)}

Reflexive – No, because every element along the diagonal is not 1.

Irreflexive – No, because every element along the diagonal is not 0.

Symmetric – Yes, because the matrix is symmetric.

Asymmetric – No, because the diagonal would need to be all zeroes and each 1 would need to correspond to a 0 when reflected across the diagonal.

Antisymmetric – No, because the ones would need to correspond to a zero when reflected across the diagonal.

Transitive – No, because for each (a,b) and (b,c) in R it would imply that (a,c) would be in R. This is not true in the case of, for example, (1,2) and (2,5). We would also see (1,5) in this set and we do not.

1. Let A = ℤ and R be the relation on A where *a R b* if and only if *a* + *b* is a multiple of 4. Determine whether R is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive. For each property, either explain why R has that property or give an example showing why it does not.

This problem is similar to Examples 5 and 9 and to Exercises 13–19 in Section 4.4 of your SNHU MAT230 textbook.3

Reflexive – No, because given any integer, (a,a) would not always be a multiple of 4 and would therefore not be in the set of R.

Irreflexive – No, because given any integer (a,a) this would sometimes be a multiple of 4.

Symmetric – Yes because for each (a,b) in the set of R, this means a+b is a multiple of R. It follows that b+a would also be a multiple of R.

Asymmetric – No because it is symmetric.

Antisymmetric – No because for the elements (a,b) and (b,a) that are within R, a would not always equal b.

Transitive – Yes because since we have all of (a,b) and (b,a) in R, it follows that all (a,c) would also be within R.